Statistical Model Choice in Archaeological Chronology Construction

Elisabetta Dixon, Bryony Moody, Caitlin Buck
School of Mathematics and Statistics, University of Sheffield @ edixon3@sheffield.ac.uk
Undergraduate Research Experience funded by the Chris Cannings bequest

1. Introduction

Phases, distinct periods of land-use, are of keen interest to archaeologists establishing the chronological construction of a site.

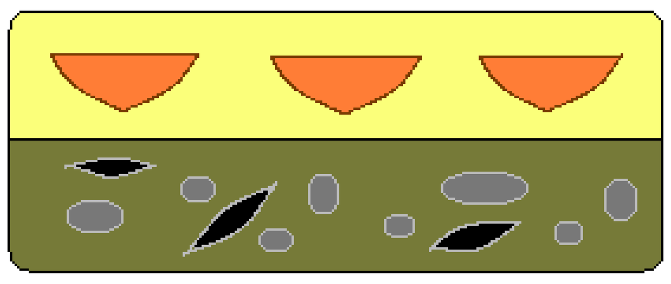

Fig. 1: Simple depiction of archaeological phases.

There are different models that estimate the **lengths** of phases from **radiocarbon dated** samples found within them. Assumptions made vary from model to model, the *ordering* of dates, the conditions applied to the *start* and *end* of phases, each producing different, even unsavoury, effects on our estimates.

2. Likelihoods

A **radiocarbon determination** is given in the form $x \pm \sigma$, where x is the lab radiocarbon date estimate and σ is the estimated error on x. To calibrate to calendar years, we use a calibration curve $\mu()$. We can assume that

$$x|\theta \sim N(\mu(\theta), \sigma^2),$$

where θ is the *true* calendar date, measured conventionally in years BP (i.e. years prior to 1950).

We use the radiocarbon **calibration curve** (with error $\delta(\theta)$) to find $L(\theta; x)$, the likelihood of θ, where

$$L(\theta; x) \propto exp\frac{-(x-\mu(\theta))^2}{2(\sigma^2+\delta(\theta)^2)}.$$

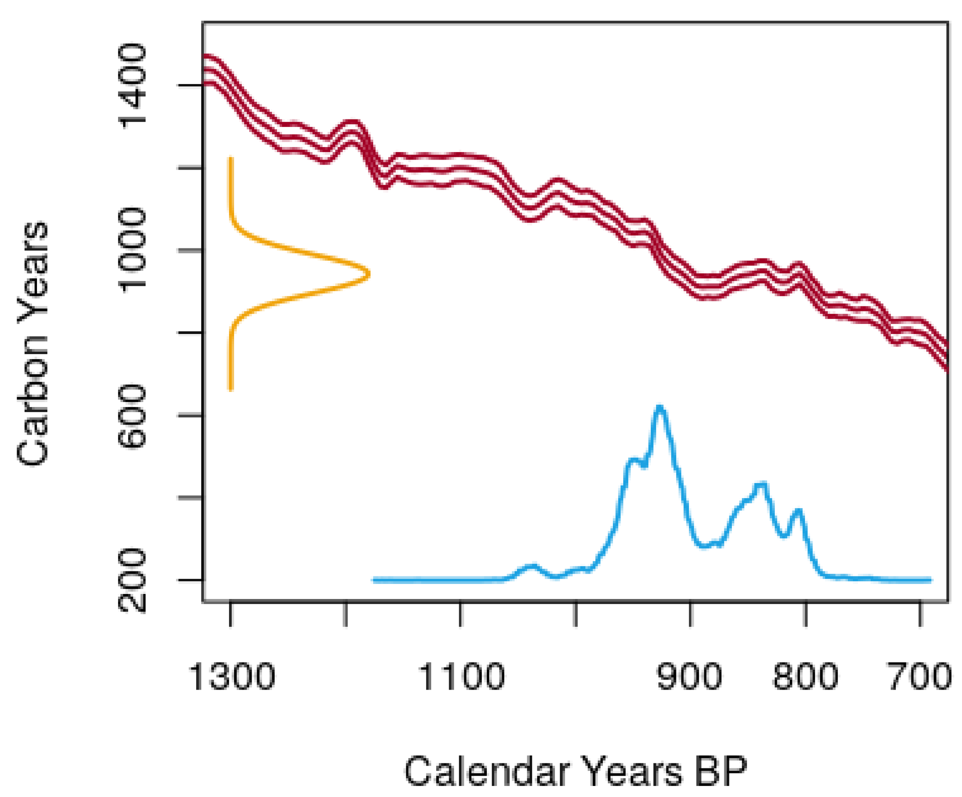

Fig. 2: Calibration of a carbon date into a calendar date range.

The multimodal likelihood produced gives a calendar date **range**, but does not make it clear where θ is most likely to lie within it.

3. Restricting date ranges

A **Highest Posterior Density** (HPD) interval is used to provide date ranges of the most likely calendar dates for θ.

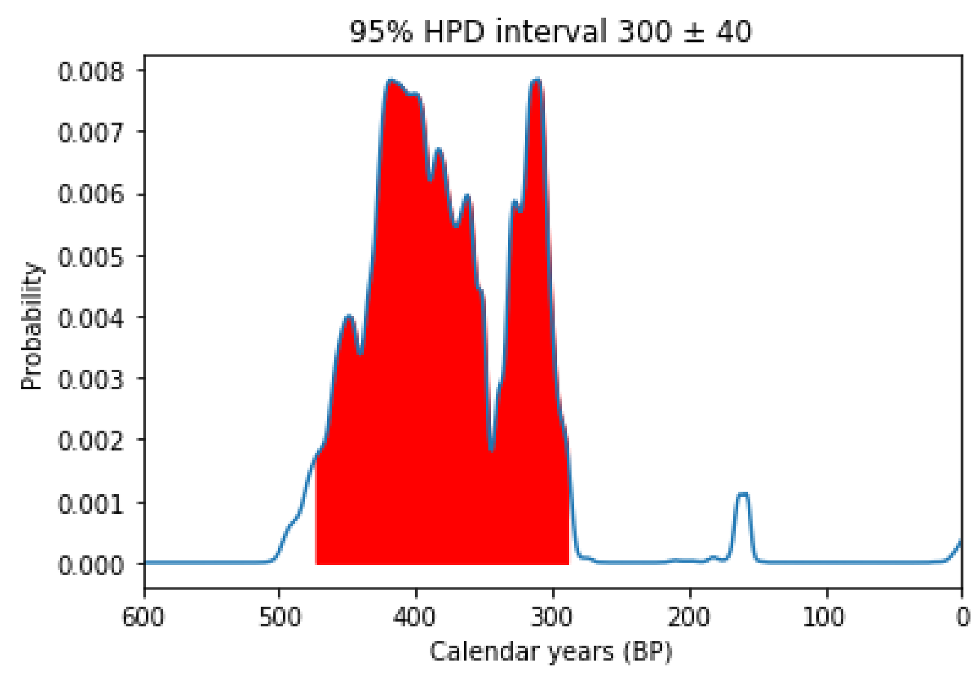

Fig. 3: The red shows the 95% HPD interval and new date range

A 95% HPD interval: the posterior probability of θ lying within the interval is 0.95, and the probability of **any** θ within the interval is **higher** than any θ outside of it.

4. Introducing multiple dates

With multiple dates, expert prior knowledge can be used to form a **prior distribution**, $P(x|\theta)$ (e.g. we *know* that of the dates θ_1 and θ_2, θ_1 is *older* than θ_2). Using Bayes theorem, we get the **posterior**:

$$P(\theta|x) = \frac{P(x|\theta)P(\theta)}{P(x)} \propto L(\theta; x)P(\theta).$$

$P(\theta|x)$, compared to $L(\theta; x)$, gives a much clearer picture of our true θ.

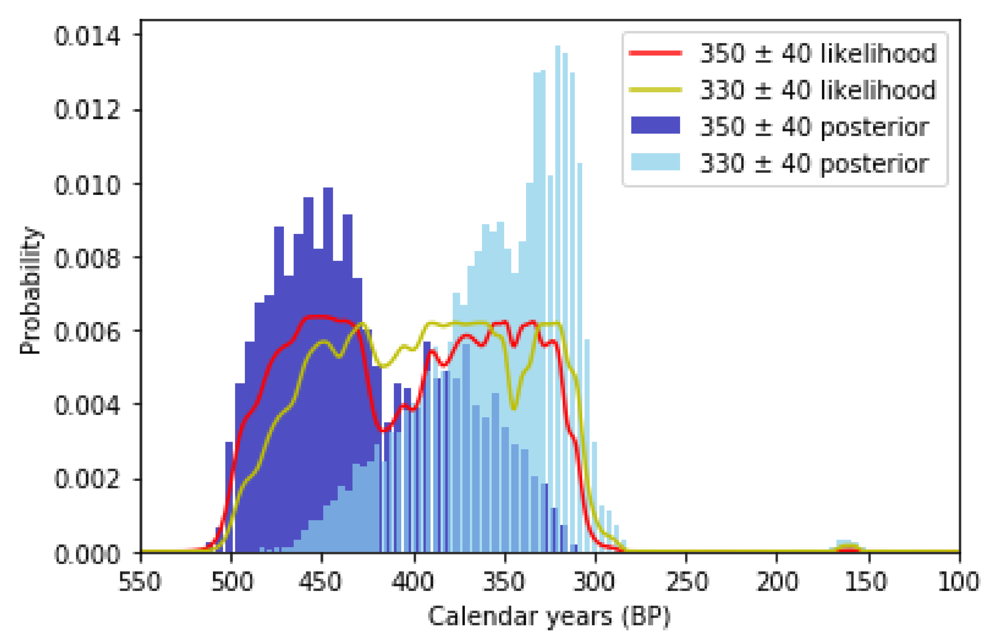

Fig. 4: Comparing likelihoods with posterior probabilities with the constraint $\theta_1 > \theta_2$

Markov chain Monte Carlo (MCMC) methods are used to estimate **marginal** posterior densities, then HPD intervals are calculated for each θ.

5. Model types

The prior knowledge used in each model impacts both the **phase length** and the **calendar date estimates**.

- **Theta model** - only uses calendar dates with no phases assumed. Phase length given as $max(\theta) - min(\theta)$
- **Alpha-beta model** - α and β are the start and end of phases respectively. Dates are uniformly distributed between α and β, with overall phase length $s = max(\alpha) - min(\beta)$. This model unfortunately favours larger values of s. See [1] for a detailed explanation.
- **Alpha-beta model with 'squeezing'**- We now assume a uniform prior distribution for s, meaning no phase length is favoured over another.

Reference
[1] Geoff Nicholls and Martin Jones. Radiocarbon dating with temporal order constraints. *Journal of the Royal Statistical Society Series C*, 50:503–521, 02 2001.

6. Data

We used the models on 3 datasets from different *time periods* and with different *phase structures*. The datasets responded similarly, but here we examine results using a dataset we **simulated**.

Our simulated data was constructed of 3 phases, each with a *true* length of 100 years.

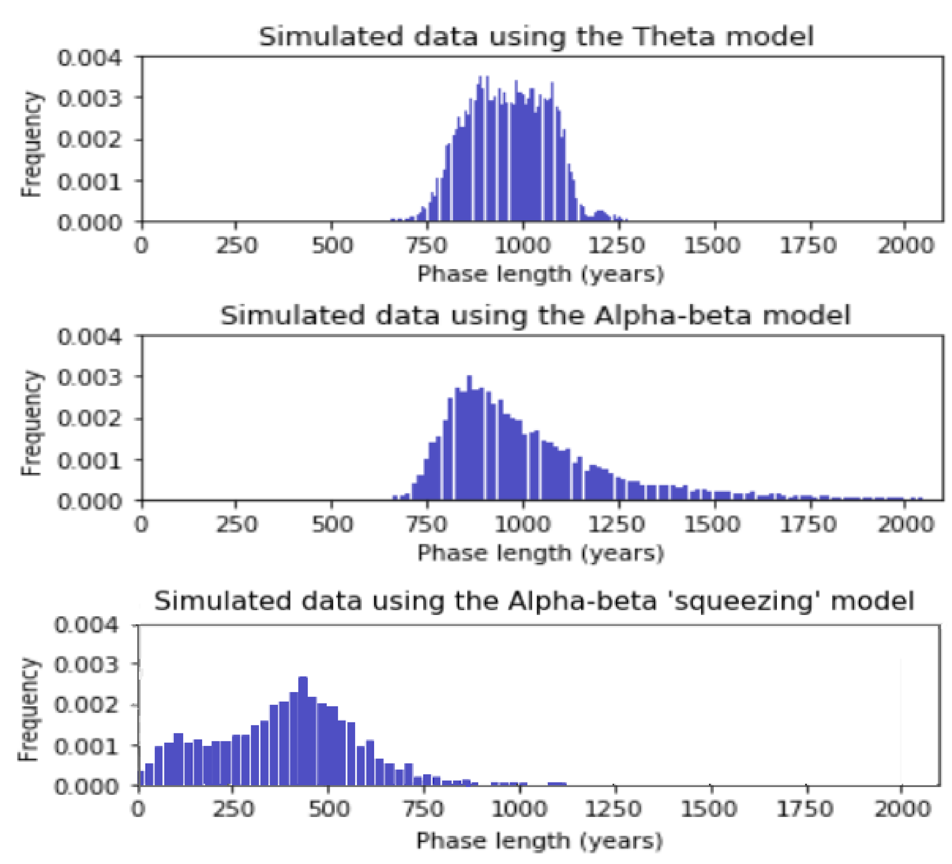

Fig. 5: Phase lengths of the Simulated data with all three models.

7. Conclusion

We see significant differences between models, and only the **Alpha-beta model with 'squeezing'** correctly predicts the phase lengths. The other two produce lengths that are hundreds of years longer, *multiple lifetimes*, thus a significant difference to an archaeologist.